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COMPLEX APPROACH TO A RADIATIVE-CONDUCTIVE HEAT TRANSFER PROBLEM IN SCATTERING SEMITRANSPARENT MATERIALS USING A DIFFUSIVE APPROXIMATION AS THE BASE

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A methodology of the analysis of combined radiative-conductive heat transfer in semitransparent strongly scattering materials is described based on simultaneous application of numerical modeling and experimental investigations.

A radiation transfer equation in combination with an energy conservation equation is commonly used for describing radiative-conductive energy transfer in absorbing and scattering media at high temperatures. An application of this approach in practice calls for a knowledge both of the radiative characteristics of boundary surfaces and of the temperature and spectral dependence of the coefficients of volume absorption k, scattering β, and volume scattering indicatrix γ for a medium (the dependence of the indicatrix on the scattered radiation direction).

However, it is impossible to employ the radiation transfer equation for describing heat transfer in scattering semitransparent porous ceramics-type materials because, first, it is very difficult to perform the numerical solution for obtaining data by the optical properties indicated above on the basis of the one-dimensional inverse radiation transfer problem even with the two-parametric description of the volume scattering indicatrix, and, secondly, in view of the fact that usually in ceramics the mean distances between structural inhomogeneities are commensurable with the quantity of inhomogeneities and with wavelength (or even less than them) and in this case the radiation transfer equation itself is invalid. The latter prevents the application of approximate description methods for radiative heat transfer (the moment method, the spherical harmonic method), based on the radiation transfer equation.

In practice, in order to describe the radiation heat transfer in scattering semitransparent materials we usually employ either the radiant heat conduction approximation or the two-flow Gurevich-Kubelka-Munck approximation. The radiant heat conduction approximation may not be associated with performance of the radiation transfer equation and here the radiative-conductive heat transfer calculation is based on the Fourier law, which it is possible to write in the form

$$q = -\Lambda_\Sigma \nabla T, \tag{1}$$

where q and ∇T are the mean-volume total flux density and the temperature gradient, while ($\Lambda_\Sigma = \Lambda_C + \Lambda_R$) is the total (conductive + radiative) thermal conductivity coefficient, defined experimentally as the temperature function through solving the inverse coefficient heat conduction problems. In such an interpretation, to describe the radiative-conductive heat transfer it is sufficient to know only one characteristic of the medium, namely, Λ_Σ, which is very tempting.

However, in scattering semitransparent materials because of the presence of the spectral region of low values for the absorption coefficient, where the single scattering albedo $\omega = \beta/(k+\beta)$ is close to unity, the local radiant equilibrium can be disturbed and the radiant thermal conductivity approximation, requiring its presence, will be not obeyed.

The two-flow Gurevich-Kubelka-Munck theory can also not be based on the transfer equation, and the radiative transfer description is rendered possible within the scope of this theory by means of two empirical parameters K and S, having the meaning of the absorption and backscattering coefficients and characterizing the

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medium optical properties. The disadvantages of this theory are the possibility of its employment for describing the heat transfer only in a plane layer in the one-dimensional formulation and the necessity of applying the same formulation during solution of inverse problems in order to define K and S. However, in the experimental determination of K and S, the two-dimensional formulation of the inverse radiative transfer problem is often preferred because it allows us to obtain a higher accuracy.

By virtue of the reasons noted above, it is advisable for describing the radiative energy transfer in porous scattering oxide ceramics and in other materials of the same class to use the diffusive approximation, based on the representation of the radiation transfer process as the photon diffusion process. With allowance for these circumstances, at the Institute of High Temperatures of the Russian Academy of Sciences a methodology has been developed for investigation of different radiative-conductive energy transfer problems; a description of it is given below.

The diffusive approximation is based on the phenomenological law

$$q = -D\nabla U, \tag{2}$$

which establishes the relation between the spectral density of the radiation flux q and the spectral volume density of radiation energy U (accurate to the divisor equal to the effective group electromagnetic wave speed). The proportionality factor D in formula (2) is the phenomenological spectral radiation diffusion coefficient, which is equal, in order of magnitude, to the mean path length of photons. For the sake of simplicity of writing, we omitted the subscripts λ indicating that the characteristics under consideration belong to a definite wavelength,

According to the aforesaid,

$$q = q^+ - q^- = \int_0^1 (I^+ - I^-)\,\mu d\mu, \tag{3}$$

$$U = 2\pi \int_0^1 (I^+ - I^-)\,d\mu, \tag{4}$$

where I is the radiation intensity; $\mu = \cos\theta$, θ is the angle characterizing the radiation propagation direction; the signs + and - specify radiation fluxes propagating forward and backward.

Here we do not touch upon the problem concerning applicability boundaries of the diffusive approximation, which has been sufficiently fully considered in [1]. We may only say that it is most acceptable for describing the radiation transfer in optically thick layers of slightly absorbing highly scattering materials, to which, specifically, we refer porous ceramics and fibrous heat insulation. For such media heated to high temperature, as was shown in [2], the macroscopic energy conservation law for monochromatic radiation may be written as

$$\operatorname{div} q = -\bar{k}U + \bar{k}\bar{n}^2 U_p, \tag{5}$$

where $U_p = 4\pi I_p$, I_p is the spectral density of the equilibrium (Planck) radiation, whereas the effective absorption coefficient k and the refraction index are associated with the absorption coefficient $\bar{k}$ and the refraction index n of the solid phase and with the porosity Π of the ceramic by the relations

$$k = (1-\Pi)\,k_0 n_0^2/[\Pi + (1-\Pi)\,n_0^2], \tag{6}$$

$$n = [\Pi + (1-\Pi)\,n_0^2]^{1/2}. \tag{7}$$

In this case the applicability condition of the diffusive approximation is defined by the relations

$$\bar{k}l \ll 1, \quad l/\Delta r \ll 1, \tag{8}$$

where l is the mean diameter of pores or grains; Δr is the minimal distance of the point considered in the material depth from the surface.

In conformity with the given above, the calculation of the radiative-conductive heat transfer in the scattering material should be performed through the simultaneous solution of the radiation diffusion equation

$$-D_\lambda(T)\,\Delta U+\bar{k}_\lambda(T)\,U_\lambda=\bar{k}_\lambda(T)\,\bar{n}_\lambda^2 U_{p\lambda}(T), \tag{9}$$

(where the subscript λ, showing the referral of the parameter to a definite wavelength, is retained) and the energy conservation equation, which for a plane layer, for example, may be written in the form

$$C_p(T)\,\rho\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\Lambda_C(T)\frac{\partial T}{\partial x}-\int\limits_{(\lambda)}\bar{k}_\lambda(T)\,(\bar{n}_\lambda^2 U_{p\lambda}(T)-U_\lambda)\,d\lambda, \tag{10}$$

where C_p is heat capacity; ρ is density; t is time. Integration in (10) is realized over the whole semitransparency region (λ). The corresponding initial and boundary conditions should be added to Eqs. (9) and (10).

The particular example of the radiative-conductive heat transfer calculation as applied to the problem of heating a plane ceramic layer by a monochromatic radiation flux can be found in [3].

The methodology of the complex approach to the solution of various problems of radiative-conductive heat transfer (RCH) in scattering semitransparent materials is schematically illustrated in Fig. 1.

The left part of the diagram refers to the solution of direct problems of RCH, whose aim is to calculate temperature fields and energy fluxes in the material. The solution of direct problems includes creation of physical and mathematical models for RCH conforming to different specific heating conditions. A mathematical model is based on the application of the radiation diffusion equation (9) and of one or another energy conservation equation of the type of equation (10) with appropriate initial and boundary conditions.

Since solutions for various problems of RCH may be obtained only by numerical methods, the important stage is the development of corresponding computational programs on a computer.

To carry out such computations, we must have data on the optical and thermal properties of the semitransparent material and the boundaries; in this case they should be presented in a form convenient for entering into a computer, for example, in the form of approximation equations. In preparation of the data, we choose peculiar spectrum regions, such as an electron absorption edge, a high-transparency region, a band edge associated with multiphonon absorption. The temperature range can also be divided into some regions related to the presence of phase transitions. The corresponding stage of data preparation for calculation of RCH is shown in Fig. 1.

The creation of the physical and mathematical models of RCH is always connected to one or another extent with application of various assumptions whose validity requires experimental verification. As shown in Fig. 1, such verification must often precede the development of programs for computation of RCH.

In connection with the fact that data on the temperature and spectral dependences of the optical properties (k and D) and the temperature dependence of the conductive thermal conductivity coefficient $_c$ may be obtained only experimentally, it is necessary to perform corresponding measurements. The choice of such inverse problems of radiative and radiative-conductive energy transfer and the development of corresponding methods for calculation of the optical properties and the thermal conductivity, which allow one to obtain the needed characteristics with the best accuracy, must precede the measurements. These investigation stages are plotted in the right part of Fig. 1.

Derivation of numerical values is associated with solution of appropriate problems on a computer. Only after obtaining the necessary initial data on the optical and thermal properties of the semitransparent material and the boundary surfaces, is it possible to perform their preparation for calculation of radiative-conductive heat transfer, to carry out the final computer software development, and to fulfill numerical computations of temperature fields and energy fluxes.

Usually, the goal of such computations consists in analyzing the serviceability of constructions of thermal nodes and units (in which semitransparent scattering materials are used) and in working out recommendations on the improvement of the composition and structure of these materials for the purpose of ensuring necessary values for the optical and thermal properties.

As noted above, the data on the optical properties are derived as a result of solving inverse radiative transfer problems. The results of measurements of the radiating capacity ε, the transmitting capacity P, or the reflecting power R can serve as information for obtaining k and D within the high transparency spectral region. It is evident from the analysis carried out by us that the reflecting power R for strongly scattering materials is slightly sensitive to a change in the absorption coefficient; therefore, we may use only the measurement results for P and ε.

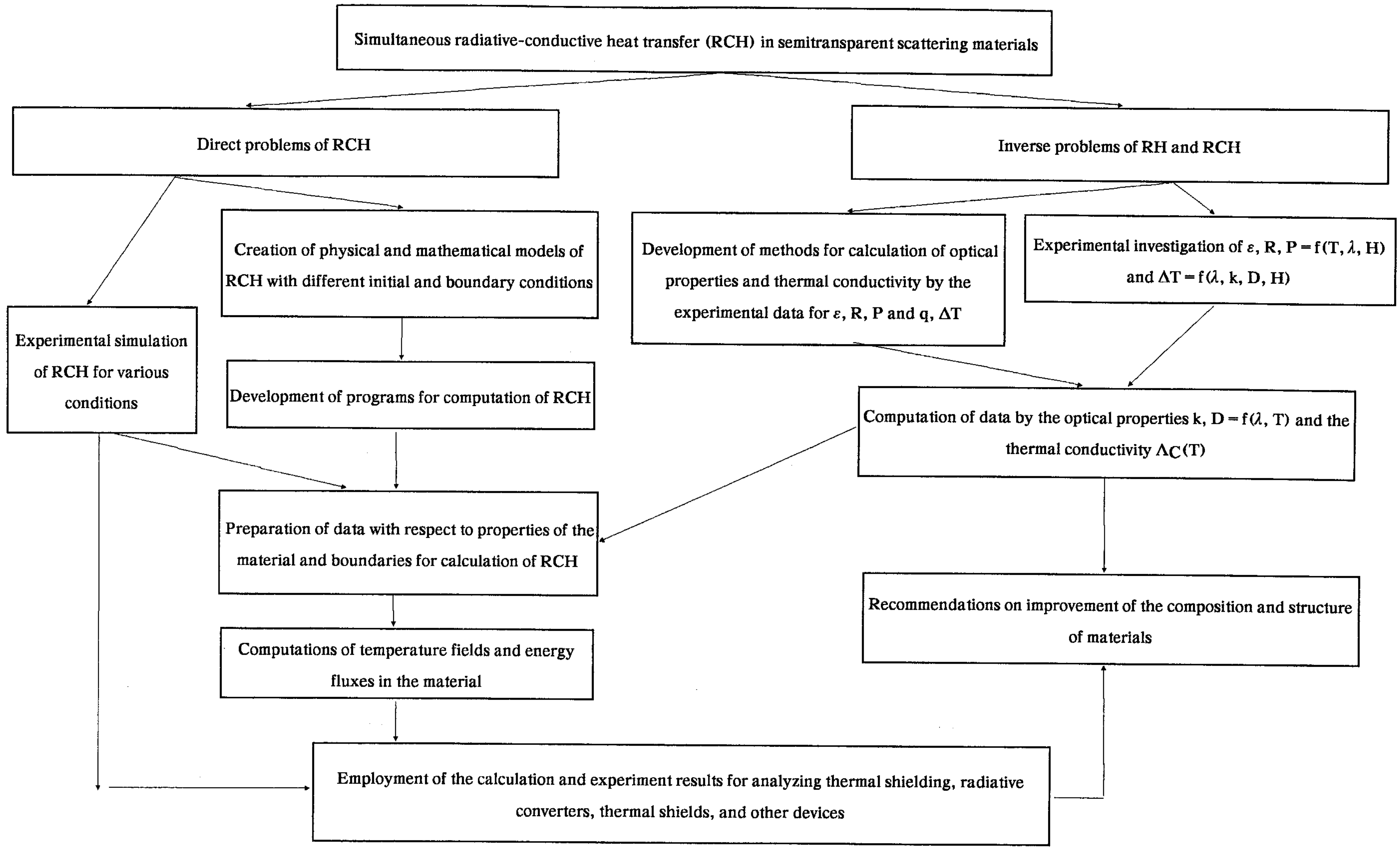

Fig. 1. Methodology of investigation of the radiative-conductive heat transfer in porous scattering semitransparent materials.

Figure 2 exhibits a diagram which makes clear the methodology of the data-obtaining with respect to the properties of the materials and preparation of them for performing computations for one of the particular examples of the RCH calculation under the conditions of the powerful optical radiation effect on the porous ceramics. The left part of the figure refers to the optical properties.

It should be noted that the data on $\bar{k}$ and D are required only for the weak-absorption spectral region. In the strong-absorption spectral region, where the radiative characteristics are determined by the properties of a very thin surface layer, the ceramic is usually characterized by the radiating capacity or the reflecting power, whose values appear in the boundary conditions.

Measurements of the thermoradiative characteristics, on the basis of which we calculate $\bar{k}$ and D, are realized by two methods.

In the temperature interval from room temperature up to about 1000 K the method is based on the measurement of the dependence of the normally hemispheric transmitting capacity P_{nh} for a set of plane samples of different thicknesses H with a cylindrical lateral surface on the wavelength and temperature, and on the measurement of the angular dependence of the directed hemispheric transmitting capacity $P_{\theta h}$ for a sample of one thickness H_0 at room temperature [4, 5]. At high temperatures it is preferable for determining $\bar{k}$ and D to use the measurement method of the dependence of the normal radiating capacity ε_n for a set of samples of different thicknesses on the wavelength and temperature, applying the data on the angular dependence of the directed hemispheric transmitting capacity $P_{\theta h}$ for one sample at room temperature. The measurement method for ε_n is based on employment of black-body furnace-model discharging and on spectrum velocity scanning [6].

Within the strong-absorption region we measure either the dependence of the spectral normal radiating capacity ε_n as a function of the wavelength and temperature by the same method or at $T \leq 1000$ K (when the radiated energy fluxes are small) the spectral normally hemispheric reflecting power on a setup intended for measuring the normally hemispheric transmission.

In addition to the optical properties, in order to calculate RCH we must know the data on the conductive thermal conductivity Λ_c of the ceramic. At low temperatures, when the radiation contribution to energy transfer is small, such data may be obtained by usual methods for measuring the thermal conductivity coefficient, but in the region of high temperatures (where radiative-conductive transfer takes place) the value of the conductive thermal conductivity coefficient Λ_c may be derived only by means of solving the inverse radiative-conductive heat transfer problem. The diagram representing the data-obtaining methodology for Λ_c is shown in the middle part of Fig. 2.

The radiative-conductive heat transfer in the porous ceramic is investigated on a setup intended for measuring the thermal conductivity coefficient, based on application of the plane temperature wave method [7]. According to such a procedure, a plane sample is placed inside a furnace and heated to the required temperature level. Then the CO_2-laser radiation, modulated by a mechanical cutter, is directed onto one of the sample surfaces. This radiation is absorbed in a very thin surface layer, and a thermal wave passes over the sample. During the experiment, we measure the phase difference C between oscillations of the laser radiation flux, impinging on the sample, and temperature vibrations on the opposite surface, as well as the amplitude of temperature vibrations A on this surface. It is possible to carry out the experiments at different frequencies ω with samples of various thicknesses H. In a rectangular modulation of the incident flux we may use harmonic Fourier analysis for studying a signal at different frequencies. In calculations of $\Lambda_c(T)$ the previously obtained data on the optical properties of the porous ceramic are employed (see the left part of Fig. 2).

The left and middle parts of Fig. 2 illustrate two necessary stages preceding the performance of calculations of the radiative-conductive heat transfer in semitransparent scattering materials. These stages are typical for very different problems, for example, for calculations of the quartz fibrous thermal shielding in orbital vehicles of repeated utilization, for calculations of turbojet engines and other devices that use heat-insulation erosion-resistant oxide coatings, sputtered by plasma spraying, and for calculations of concentrated solar radiation receivers with a porous ceramic insert.

The right part of Fig. 2 and its middle part (which is related to the melt) show the stage important for calculation of RCH under the conditions of the influence of intense optical radiation a porous oxide ceramic when an ablating melt layer is formed on the surface. The same situation can take place during intense convective heating of

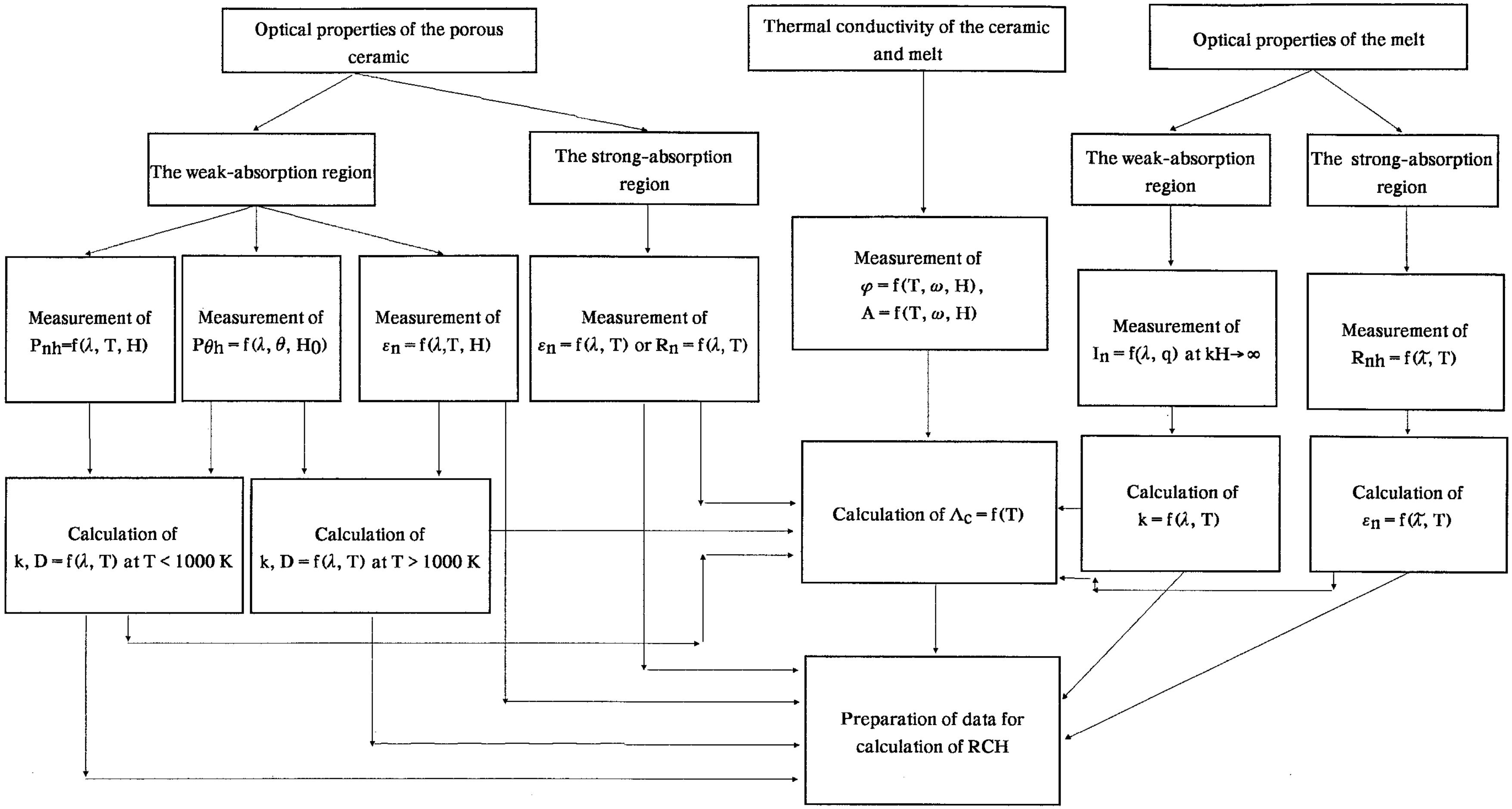

Fig. 2. Diagram of obtaining and preparation of data on the properties of a porous ceramic for calculation of RCH under the action of powerful radiation fluxes upon it.

the destroyed ceramic thermal shielding of spacecrafts entering dense atmospheric layers. In the all these cases, in addition to the optical properties and the conductive thermal conductivity coefficient of the ceramic, it is necessary to know also the optical and thermal properties of the melt. We may determine Λ_c for the melt by the same temperature wave method which is used for defining the thermal conductivity coefficient of the ceramic. However, it is impossible, of course, to determine the optical properties of the melt by the methods employed for defining the optical properties of the ceramic. In this case some special techniques are needed. Not dwelling upon them more fully, we note the main difficulty. When determining the optical properties of the melt in the weak-absorption region, it is virtually impossible because of high temperatures to keep the melt in any cuvettes with transparent windows since a chemical interaction between the melt and the window material can occur. The melt may be investigated only in the form of a puddle formed by local melting of the ceramic. As the characteristics of the optical properties of the melt within the weak-absorption region, we take the absorption coefficient and the refraction index. In view of the fact that the refraction index during melting usually does not vary strongly, whereas the requirements to its accuracy for calculations of RCH do not exceed as a rule several percent, it is often possible to ignore measurements of the refraction index of the melt and to define it only for the solid phase and to make a correction for the change in the density during melting. Therefore, in the weak-absorption region we usually measure only the dependence of the absorption coefficient on the wavelength and temperature, and practically the only measured parameter (which can be used for calculations of k) is the radiation spectrum of such a melt. In order to exclude the effect of the solid phase on the radiation spectrum, the melt layer should be optically infinite. In this connection the radiation intensity I of the optically infinite melt layer for every wavelength is determined both by the dependence of the absorption coefficient on temperature and by the temperature field inside the melt. The definition of the dependence $k = f(\lambda, T)$ may be carried out via the solution of the inverse radiation transfer problem simultaneously with the determination of the temperature field distribution inside the melt. In this case we experimentally measure the dependence of the radiation intensity in the direction normal to the surface $I = f(\lambda, q)$ on the wavelength at different q under quasi-stationary conditions, i.e., at various levels of the surface temperature. The other formulation of inverse problems is also possible.

In the strong-absorption region of the melt (the nontransparency region) the normally hemispheric reflection power R_{nh} is measured as a function of the wavelength and temperature.

The resulting data on the optical properties of the melt are employed for calculation of its thermal conductivity coefficient Λ_c as well as for calculations of temperature fields and energy fluxes under conditions of the effect of intense radiation on the porous oxide ceramic.

In conclusion it may be noted that the presented complex methodology for investigation of radiative-conductive heat transfer in a porous scattering ceramic was developed during the last six years at the Institute of High Temperatures of the Russian Academy of Sciences. For now not all the works are realized in completed form but there are grounds to believe that at present we have accomplished the fundamental and most tedious part of the technique, connected with creation of the experimental basis and with elaboration of the computer software for solving direct and inverse problems.

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